MAXIMUM ENTROPY IMAGING WITH COMPTEL DATA

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Abstract

COMPTEL is the Compton telescope on board NASA's Gamma Ray Observatory due to be launched in 1990. A prime objective is complete sky mapping of celestial gamma rays in the 1-30 MeV energy range. Owing to the complex response of Compton telescopes, conventional imaging techniques are inadequate to produce maps which correctly make use of the full response function. The three-dimensional data space consisting of photon scatter direction and Compton scatter angle is difficult to treat by normal methods. For these reasons we have decided to use the maximum entropy method as a principal imaging technique for COMPTEL. Since this method allows explicit inclusion of the instrumental response to any given input intensity distribution, it is able to produce maps which are true deconvolutions of the data.

Introduction Compton telescopes provide one of the few techniques for performing γ-ray astronomy with $\approx 1^o$ angular resolution in the 1 - 30 MeV range. So far Compton telescopes have been flown on only balloons (e.g. the MPE Compton Telescope, Graser and Schönfelder 1982, von Ballmoos et al. 1987a,b, and the Riverside group, e.g. O'Neill et al. 1989). Because of the forthcoming launch of NASA's Gamma-Ray Observatory, with COMPTEL on board, it is important to study methods to obtain the best possible images for scientific analysis.

A Compton telescope detects the Compton scattering interaction of the incoming photon in an (upper) plane of detector elements, followed by absorption of the scattered photon in another (lower) plane of highly absorbing detector material. The simplest way to generate an 'image' from Compton telescope data is the so-called 'event-circle' method. The combination of measured scatter direction and scatter angle results in an 'event circle', which contains the true photon arrival direction in the ideal case. Regarding the circles as measures of the probability distribution on the sky for each event, summation of circles for many events results in an 'image'. The first skymap of the Galactic anticentre in the MeV range was generated with this method (Graser and Schönfelder 1982). Later von Ballmoos et al. (1987a,b) improved the event filtering techniques, applied systematic corrections (derived from Monte-Carlo calculations) to the 'event circle' parameters, and combined the individual event's information into an image via a likelihood product. Thus they were able to derive images of the Galactic Centre region and the nearby active galaxy Cen A. However the 'image' produced by the event-circle method does not satisfy the key criterion: folded with the instrumental response it should be consistent with the observed data.

The maximum entropy method is an obvious choice for imaging since it *does* give an image consistent with the data; further it requires only that the detector response to any given image can be computed. This method is especially useful where the instrument response is such that no one-to-one relation exists between individual image and data bins, so that there is no 'conventional' method which can produce an image at all - such is the case for the Compton telescope. The reader is referred to Gull and Skilling (1984) for an explanation of the principles of the maximum entropy method. A brief review of γ-ray applications is given by Strong and Diehl (1989).

Characteristics of COMPTEL Data The characteristics of GRO-COMPTEL are summarized below: (see also Schönfelder et al. 1984).

Energy range	1-30 MeV
Field of view	60°
Sensitive area	20–40 cm^2
Energy resolution	5-8%
Angular resolution FWHM	2.7°(4 MeV)–4.7° (1 MeV)
Expected mission duration	2 to >5 years

The prime measurement quantities of an event in a Compton telescope are the energy deposits in the two detector assemblies, and the position of the interactions in both planes. In the COMPTEL telescope there are 7 upper detector modules of liquid scintillator and 14 lower detector modules of NaI crystal, the two planes being separated by 1.5 m. In both detector module types the positional measurement is made by comparing the relative pulse heights of all individual photomultiplier tubes of the corresponding module.

The Compton scatter process in the upper detector plane determines the particular directional information which is measured by the instrument for each photon; because of the probabilistic nature of this scatter process a wide range of scattering angles is possible for photons incident on the instrument with the same angles and energies. Therefore each pixel in image-space is translated into a multi-parameter distribution in data-space.

The measurement process in a Compton telescope for a mono-energetic input sky can be described by

$$n(\vec{x}_1, \vec{x}_2, E_1, E_2) = \int\int R(E_\gamma, \chi_o, \psi_o; \vec{x}_1, \vec{x}_2, E_1, E_2) I(E_\gamma, \chi_o, \psi_o) d\chi_o d\psi_o$$

where n represents the data-space distribution resulting from an image intensity distribution I, R is the response function, $\vec{x}_1, \vec{x}_2$ are the (2-D) coordinates of the interactions, and E_1, E_2 the energy deposits in the upper and lower detectors, $(E_\gamma, \chi_o, \psi_o)$ represents the energy and direction of the incoming photons. Since $\vec{x}_1 - \vec{x}_2$ defines the scatter direction (χ, ψ), the number of independent dataspace parameters can to be reduced from 6 to 4; with further change of variables to scatter angle and total energy:

$$n(\vec{x}_1, \vec{x}_2, E_1, E_2) \rightarrow n(\chi, \psi, \bar{\varphi}, E_T)$$

where

$$E_T = E_1 + E_2$$

and

$$\bar{\varphi} = arccos(1 - \frac{mc^2}{E_2} + \frac{mc^2}{E_T})$$

(mc^2=.511 MeV), from the physics of Compton scattering. Thus for intervals of input photon energies E_γ, the angular response of the Compton telescope can be represented as a probability distribution in the 3-dimensional dataspace of Compton scatter angle and scatter direction $(\chi, \psi, \bar{\varphi})$.

The idealized measurement of a Compton scatter interaction, in which there is total absorption of the scattered photon in the lower detector plane, results in a pattern of allowed datapoints per image pixel which lies on a cone in $(\chi, \psi, \bar{\varphi})$-space, where the cone apex is at (χ_o, ψ_o) and the cone semi-angle is 45°. The response density along the cone is given by the variation of the Klein-Nishina cross-section for Compton scattering, which yields e.g. a falloff by 0.14 between 20° and 40° scattering for a 5 MeV photon. This idealized 'cone-mantle' response is blurred by measurement inadequacies in the scintillator detectors, in particular incomplete absorption in D2. The blurring function in dataspace, however, is relatively insensitive to the direction of the incoming photon. Therefore the angular response of a Compton telescope can be represented by

$$n(\chi, \psi, \bar{\varphi}) = g(\chi, \psi) \cdot \int\int I(\chi_o, \psi_o) A(\chi_o, \psi_o) f(\chi - \chi_0, \psi - \psi_0, \bar{\varphi}) d\chi_o d\psi_o$$

where $I(\chi_o, \psi_o)$ is the infalling intensity distribution, $A(\chi_o, \psi_o)$ is an exposure factor, $f(\chi - \chi_0, \psi - \psi_0, \bar{\varphi})$ is the 'cone' function, and $g(\chi, \psi)$ is a geometrical function accounting for the incomplete coverage of the upper and lower planes by the detectors. This representation of the response simplifies the convolution of the input sky model with the PSF sufficiently to allow use of a Fast Fourier convolution method, at least for restricted image sizes.

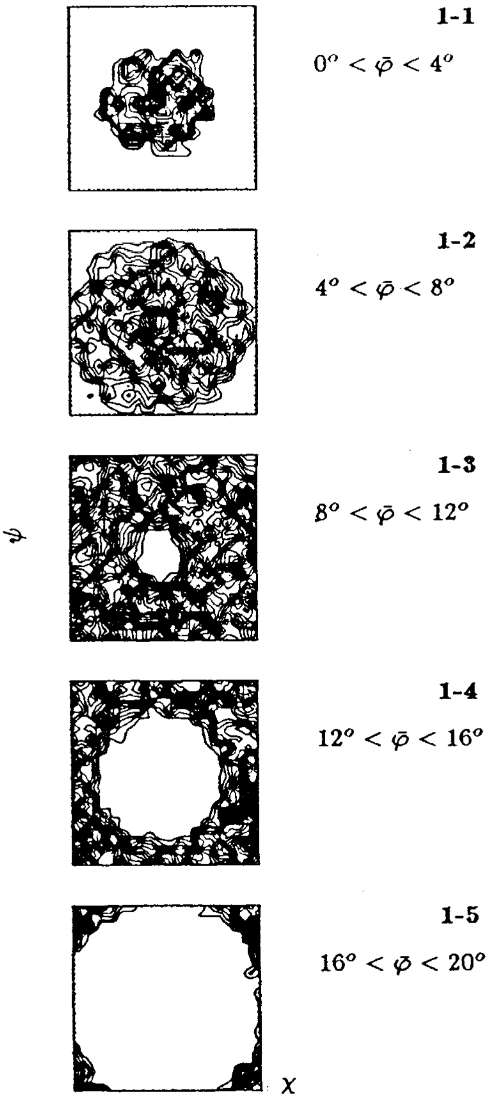

FIG 1: Simulated data in COMPTEL data space for an image consisting of seven point sources without background. The corresponding maximum entropy image is shown in Fig 2 .

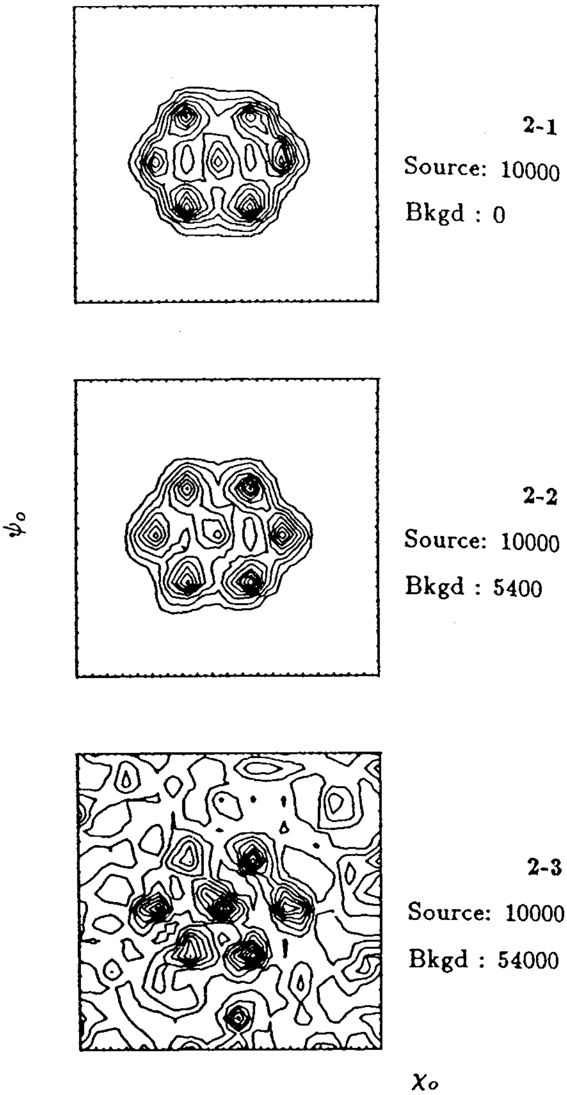

FIG 2: Maximum entropy images from data generated using a pattern of seven point sources, with varying amounts of background. The counts originating in source and background are shown next to the corresponding Figures.

<u>Application to simulated COMPTEL data</u> The maximum entropy imaging approach is to convolve a (2-dimensional) image with the full response of the telescope in the 3-dimensional dataspace $(\chi, \psi, \bar{\varphi})$, and to try to match the 'mock data' from the trial image to the measured data in the 3-dimensional dataspace. The trial image yielding a statistically acceptable match to the measured data, and at the same time fulfilling the entropy criterion, is defined to be the maximum entropy image. We use the MEMSYS package (MEDC Ltd, Cambridge) to locate the maximum entropy solution.

An example of simulated data for a single 'mini-telescope' consisting of a single upper and lower detector module pair is shown in Fig 1. There are 20 bins of 1° in χ and ψ and 5 of 4° in $\bar{\varphi}$. The (Gaussian) half-thickness of the point spread function cone was taken as 1°, to simulate a possible

COMPTEL response. The test image is an array of 7 point sources separated by 3°. The whole image is 20° square. We investigated the deconvolution of various test datasets from this image using the maximum entropy method (Fig 2). Examples of results for various background intensities are shown, varying from zero background to an intensity per (1°) pixel corresponding to 1/10 the flux from one of the point sources.

For case 1, the deconvolution is practically perfect, as would be expected in the absence of background. Case 2, with approximately equal source and background counts still resolves all sources; the 'halo' in the region between the sources indicates the problems induced by background data. Case 3 again shows all the sources, so that such an image would provide still a good basis for identification of regions of interest; the unreal structures, with up to one third of the intensity of a single source, demonstrate the limitation caused by low signal-to-noise.

Application to COMPTEL calibration data An exploratory application of the method to calibration data collected in 1987 (see Much et al. these proceedings OG10.1-10) has been made. Three calibration runs with an Na^{24} source at zenith angles separated by 20° were superimposed to simulate a dataset with three sources in the field; the maximum entropy method was then applied with an energy window selecting the 2.75 MeV line of the source. Fig 3 (from Much 1988) shows the resulting map. The three sources are successfully resolved, but this example does not show an optimized performance since it is limited by the wide (3°) binning applied in order to cover the full field.

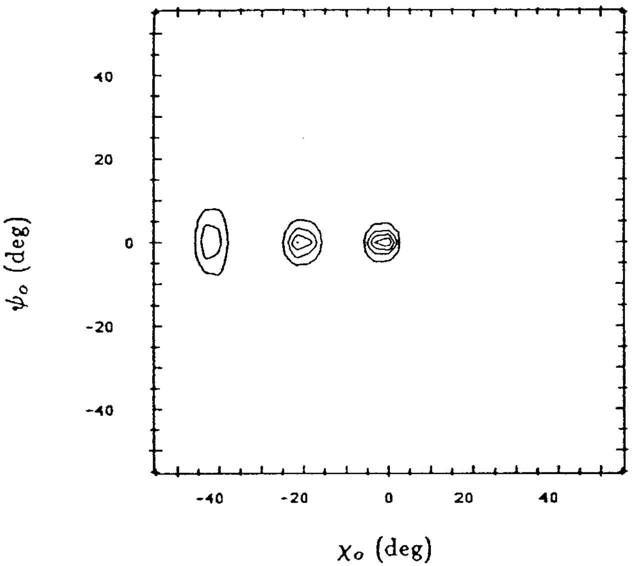

FIG 3 Image from calibration data containing 3 Na^{24} sources. Binsize is 3°. Background subtracted.

The power of the method depends critically on the prior knowledge about response and background; our investigations indicated that the background contributions in dataspace are most critical. Realistic simulations of the in-flight situation will require more knowledge of the actual background situation, which will not be known until flight data from the NASA Gamma Ray Observatory are in hand.

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